Large tensor spectrum of BICEP2 and its implications

Bumseok KYAE
(Pusan Nat'l Univ.)

arXiv: 1404.7855, PRD90 (2014) 023536,
arXiv: 1404.3756, PLB735 (2014) 391,

[arXiv: 1109.4245, PLB706 (2012) 243]

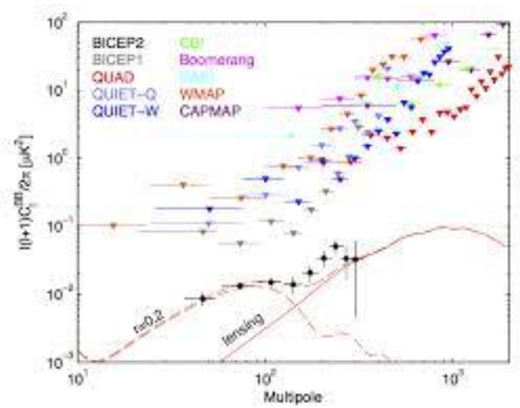
in collaboration with Ki-Young Choi (KASI)

Oct. 28 in 2014 @ The 4th KIAS workshop on particle physics and cosmology

News from BICEP2

Recently BICEP2 announced the large tensor-to-scalar ratio:

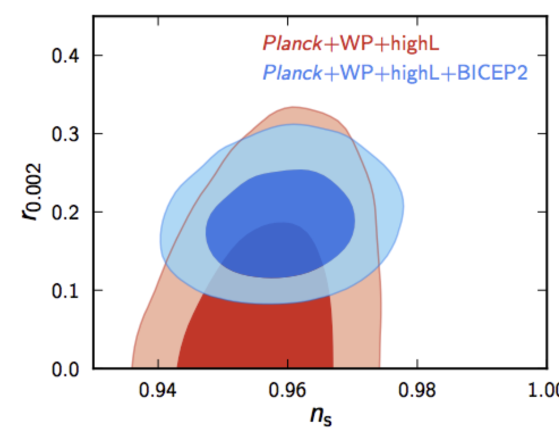
$$r = 0.2^{+0.07}_{-0.05} \quad (\text{or} \quad 0.16^{+0.06}_{-0.05}\ )$$



(after foreground subtraction with the best dust model)

Planck observation:

$$\mathcal{P}_\zeta = (2.198 \pm 0.056) \times 10^{-9}$$
$$n_\zeta = 0.9603 \pm 0.0073$$



News from BICEP2

$$r = 0.2^{+0.07}_{-0.05} \quad (\text{or} \quad 0.16^{+0.06}_{-0.05} \;)$$

The **single field** inflationary scenario with **r=0.16** implies

[$P_\zeta = V / 24\,\pi^2 M_P^4\, \varepsilon^*$, $n_\zeta - 1 = -6\,\varepsilon^* + 2\,\eta^*$, $r = 16\,\varepsilon^*$]

$$\epsilon^* \approx 0.01 \quad \text{and} \quad V^{1/4} \approx 2.08 \times 10^{16}\,\text{GeV}.$$

Super-Planckian Inflation ??

[Lyth, PRL '97]
[Antusch, Nolde, JCAP 1405 035, arXiv:1404.1821]

$$\frac{\Delta\varphi}{M_P} \gtrsim \mathcal{O}(1) \times \left(\frac{r}{0.1}\right)^{1/2},$$

Large Field Inflation?

It might imply
Breakdown of Q. F. T. description
on inflation.

Super-Planckian Inflation ??

$$\Delta N \approx \frac{1}{M_P}\int \frac{d\varphi}{\sqrt{2\epsilon}} \approx 7\left(\frac{\Delta\varphi}{M_P}\right)\sqrt{\frac{0.01}{\epsilon}}.$$

For $\varepsilon = 0.01$ and $\Delta\varphi \sim M_P$,

$\Delta N \sim 7$!!

To be consistent with the observation on CMB,
Power Spectrum should be almost constant
for 10 Mpc < k^{-1} < 10^4 Mpc, i.e. for $\Delta N \sim 7$.

Super-Planckian Inflation ??

$$\Delta N \approx \frac{1}{M_P} \int \frac{d\varphi}{\sqrt{2\epsilon}} \approx 7 \left(\frac{\Delta\varphi}{M_P}\right) \sqrt{\frac{0.01}{\epsilon}}.$$

For $\varepsilon = 0.01$ and $\Delta\varphi \sim M_P$,

$\Delta N \sim 7$!!

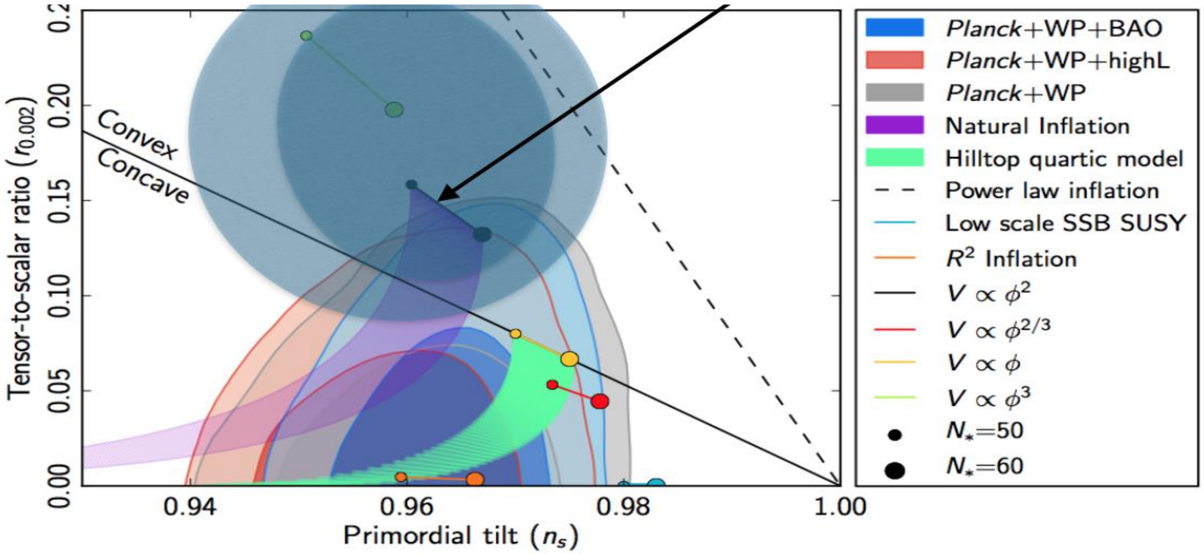
Sub-Planckian field variation

yields a too small e-folding number (<< 50)

in **single** field inflation.

Chaotic Inflation favored !?

$V = (m^2/2)\,\phi^2$



$\varepsilon^* \approx 0.01 \leftrightarrow \langle\phi\rangle \approx 15\, M_P$

$V^{1/4} \approx 10^{16}\, \text{GeV} \leftrightarrow m \approx 10^{13}\, \text{GeV}$

Chaotic Inflation favored !?

$$V \supset (m^2/2)\,\phi^2\,[1 + c_1(\phi^2/M_P{}^2) + c_2(\phi^4/M_P{}^4) + \ldots]$$

Perturbation would break down !!

while

$$L/(-g)^{1/2} \supset (M_P{}^2/2)\,R\,[1 + c_1 R/M_P{}^2 + c_2 R^2/M_P{}^4 + \ldots]$$

is still O.K.

↑

$\varepsilon^* \approx 0.01 \leftrightarrow \langle\phi\rangle \approx 15\,M_P$

$V^{1/4} \approx 10^{16}$ GeV $\leftrightarrow$ $m \approx 10^{13}$ GeV

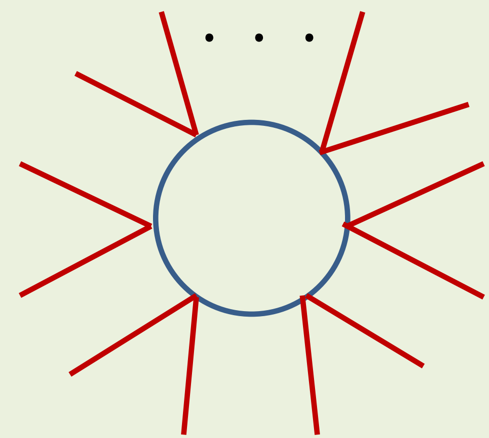
Chaotic Inflation favored !?

$$V \supset (m^2/2)\,\phi^2\,[1 + c_1(\phi^2/M_P{}^2) + c_2(\phi^4/M_P{}^4) + \ldots]$$

Perturbation would break down !!

L/ ..]

- e.g. $L \supset g\,g\,(\partial\Phi)^2/M_P{}^2$



- $L \supset M_P{}^4(\Phi/M_P)^{2n}$ might be generated also by strongly coupled gravity interaction.

We should accept the **paradigm of super-Planckian Inflation.**

Or we need

A **Two** (or **Multi-**) **Field** **Inflationary Model** !!

For a naturally light scalar field (Inflaton)

Only a few NATURAL ways to get a light scalar are known in Q.F.T. :

By introducing

1. Supersymmetry (χ-sym.-respecting superpartner)

OR

2. A global symmetry (pseudo-Goldstone boson)

OR

3. A strong dynamics (composite scalar)

For a naturally light scalar field (Inflaton)

Only a few NATURAL ways to get a light scalar are known in Q.F.T. :

By introducing

1. Supersymmetry $\longrightarrow$ "SUSY hybrid Inflation"

   OR

2. A global symmetry $\longrightarrow$ "Natural Inflation"

   OR

3. A strong dynamics

“η Problem” in SUGRA

Suppose that $V_F = \sum_i |\partial_{\varphi i} W|^2 \equiv \Lambda$ in global SUSY.

In SUGRA,

$$V_F = \mathrm{Exp}[K/M_P] \left[(D_{\varphi i}W)(K^{-1})^{ij}(D_{\varphi j}W)^* - 3|W|^2/M_P^2 \right]$$
$$\approx \mathrm{Exp}[K/M_P]\, \Lambda$$

where $D_{\varphi i}W \equiv \partial_{\varphi i}W + W\partial_{\varphi i}K/M_P^2$

“η Problem” in SUGRA

Suppose that $V_F = \sum_i |\partial_{\varphi i} W|^2 \equiv \Lambda$ in global SUSY.

In SUGRA,

$$V_F = \mathrm{Exp}[K/M_P] \left[(D_{\varphi i}W)(K^{-1})^{ij}(D_{\varphi j}W)^* - 3|W|^2/M_P^2 \right]$$
$$\approx \mathrm{Exp}[K/M_P]\ \Lambda$$

For $K = |\varphi|^2 + \ldots$, $V_F \approx \Lambda + (\Lambda/M_p^2)|\varphi|^2 + \ldots$

$\rightarrow \eta = 1$!!

I. SUSY Hybrid Inflation

Introduce $U(1)_R$ sym.

[Copeland etal. '94]

$$W \rightarrow e^{2i\gamma} W \quad ; S \rightarrow e^{2i\gamma} S$$

$$K = |S|^2 \quad ; \quad W = Sm^2$$

("K" is the minimal Kahler pot., and "W" is of the "Polonyi" type.)

"S^2", "S^3", etc. don't appear in W !!

I. SUSY Hybrid Inflation

$$D_S W = m^2(1 + |S|^2)\ ,\ K_{SS} = K^{SS} = 1\ ;$$

$(M_P{}^2 \equiv 1)$

$$V_F = e^K\ [\ |D_S W|^2 - 3|W|^2]$$
$$\approx m^2\ [1 + 0 + (1/2)|S|^4 + \ldots]$$

The Hubble scale mass term is accidentally cancelled!

Let us introduce also the waterfall fields ψ , ψ^c .

$$W = S\,(m^2 - \psi\psi^c)\,;$$

$$V = |m^2 - \psi\psi^c|^2 + |S|^2(|\psi|^2 + |\psi^c|^2)$$

At SUSY minimum, $S = 0$, $\psi\psi^c = m^2$

But if $S \gg m$, then $\psi = \psi^c = 0$ and

$W_{eff} = S\ m^2 \ \longrightarrow \ V = m^4$: semi-stable false vacuum

By including the quantum correction, [Dvali, Shafi, Schaefer, '94]

$$V_{inf} = m^4\,[\,1 + (1/8\pi^2)\,\mathrm{Log}\,(S/\Lambda)\,]$$

A problem in SUSY Hybrid Infl.

Prediction: $n_\zeta \approx 1 + 2\eta = 1 - 1/N_e = 0.98$

for $N_e = 55\text{-}60$,

while data of WMAP7 and PLANCK give $n_\zeta \approx 0.96$.

$N_e = 25$ for $n_\zeta \approx 0.96$ is not enough.

II. Natural Inflation in SUGRA

Introduce a shift sym.

[Freese etal. '90]

[Kawasaki etal. '00]

$$T \longrightarrow T + 2\pi i f \qquad (\text{ i.e. } a \rightarrow a + 2\pi f, \text{ axion })$$

$$K = K(T+T^*) \quad \text{or} \quad K = K(\mathrm{Re}T)$$

$$W = w_0 + m^3 e^{-T/f}$$

"a" doesn't appear in Kahler potential !!

A problem in Natural Infl.

$$V_{inf} = \Lambda^4 [\, 1 - \cos(\, a/f\,)\,]$$

[e.g. by instnaton effect]

$f > 3\, M_P$ for $\eta \ll 1$,

where f is the $U(1)_{PQ}$ breaking scale.

$U(1)_{PQ}$ above the quantum gravity scale ?

Again,

We need

a Two (or Multi-) Field Inflationary Model !!

For a naturally light scalar field (Inflaton)

- Natural inf. + Natural inf. = Kim-Nilles-Peloso '05 ($f < M_P$)
- Natural inf. + SUSY hybrid inf. = "Natural hybrid inf." ($f < M_P$, n_ζ=0.96)
- SUSY hybrid inf. + SUSY hybrid inf. = "Double hybrid inf." (n_ζ=0.96)

1. Supersymmetry → "SUSY hybrid Inflation"

   OR

2. A global symmetry → "Natural Inflation"

   OR

3. A strong dynamics

For a naturally light scalar field (Inflaton)

- Natural inf. + Natural inf. = Kim-Nilles-Peloso '05 ($f < M_P$)
- √ Natural inf. + SUSY hybrid inf. = "Natural hybrid inf." ($f < M_P$, n_ζ=0.96)
- SUSY hybrid inf. + SUSY hybrid inf. = "Double hybrid inf." (n_ζ=0.96)

1. Supersymmetry → "SUSY hybrid Inflation"

   OR

2. A global symmetry → " Natural Inflation"

   OR

3. A strong dynamics

"Natural-Hybrid Model"

[Choi, BK '12]

With both $U(1)_R$ symmetry and a shift symmetry, under which the two inflatons transform as

$$S \longrightarrow e^{2i\gamma} S \ , \quad T \longrightarrow T + 2\pi i f \ ,$$

where $T = \phi + i\,a$

$$W = \kappa S \left(M^2 - \sum_n m_n^2 e^{-nT/f} - \psi\psi^c \right)$$

$$K = |S|^2 + (1/2)(T + T^*)^2 + |\psi|^2 + |\psi^c|^2$$

The kinetic terms of the inflatons are of the canonical type:

$$L_{kin} = |\partial S|^2 + (1/2)[(\partial\phi)^2 + (\partial a)^2]$$

“Natural-Hybrid Model”

Goal

1. Achieve $r = 0.16$, $n_\zeta = 0.96$
2. With $f < M_p$
3. Obtaining a large enough N_e

by combining the SUSY hybrid and Natural inflation models

"Natural-Hybrid Model"

$$V/\kappa^2 = |M^2 - m^2 e^{-T/f} - \psi\psi^c|^2$$
$$+ |S|^2 [\, |\psi|^2 + |\psi^c|^2 + (m^4/f^2)\, e^{-2\phi/f}\,]$$

Assume $m^2/M^2 \ll f^2/M_{pl}^2 < 1$

At SUSY minimum, $M^2 - m^2 e^{-T/f} - \psi\psi^c = 0\,,\ S = a = 0$

By including soft terms, $\phi/f \sim O(m^2/f^2) \ll 1\,,$ so $M^2 - m^2 \approx \psi\psi c$

“Natural-Hybrid Model”

$$V/\kappa^2 = |M^2 - m^2 e^{-T/f} - \psi\psi^c|^2$$
$$+ |S|^2 [\, |\psi|^2 + |\psi^c|^2 + (m^4/f^2)\, e^{-2\phi/f} \,]$$

For inflation,

initially $S > M$

$a/f \approx 0.4$

Then during inflation,

$\psi = \psi^c = 0$, and

$W_{eff} = \kappa S\, (M^2 - m^2 e^{-T/f})$

By including SUGRA correction (Hubble induced mass term),

$\phi/f \sim O(m^2 M_{pl}^2 / f^2 M^2) \ll 1$

But “S” and “a” can be inflatons against SUGRA corrections.
i.e. No Hubble induced mass terms for them.

“Natural-Hybrid Model”

During inflation, the waterfall fields are decoupled, and so the superpotential and Kahler potential are

$$W_{\text{inf}} = \kappa S \left(M^2 - \sum_n m_n^2 e^{-nT/f} \right), \quad \text{and} \quad K_{\text{inf}} = |S|^2 + \frac{1}{2}(T + T^*)^2,$$

We require

$$\frac{m_n^2}{M^2} \ll \frac{f^2}{M_P^2} \lesssim 1.$$

“Natural-Hybrid Model”

During inflation, the waterfall fields are decoupled, and so the superpotential and Kahler potential are

$$W_{\rm inf} = \kappa S \left(M^2 - \sum_n m_n^2 e^{-nT/f} \right), \quad \text{and} \quad K_{\rm inf} = |S|^2 + \frac{1}{2}(T+T^*)^2,$$

$$\begin{aligned}
V_{\rm SUGRA} &= e^{K/M_P^2}\left[|D_S W|^2 + |D_T W|^2 - 3\frac{|W|^2}{M_P^2} \right] \\
&\approx \kappa^2 M^4 \left(1 + \frac{\phi^2}{M_P^2}\right) \left[\left| 1 - \sum_n \frac{m_n^2}{M^2} e^{-nT/f} \right|^2 + \frac{f^2 |S|^2}{M_P^4} \left| \frac{\sqrt{2}\phi}{f} + \frac{M_P^2}{f^2} \sum_n n \frac{m_n^2}{M^2} e^{-nT/f} \right|^2 \right] \\
&\approx \kappa^2 M^4 \left[1 - \left(\sum_n \frac{m_n^2}{M^2} e^{-nT/f} + \text{h.c.} \right) + \frac{\delta\phi^2}{M_P^2} \right],
\end{aligned}$$

where $(T+T^*) = \sqrt{2}\phi$

“Natural-Hybrid Model”

During inflation, the waterfall fields are decoupled, and so the superpotential and Kahler potential are

$$W_{\rm inf} = \kappa S\left(M^2 - \sum_n m_n^2 e^{-nT/f}\right), \quad \text{and} \quad K_{\rm inf} = |S|^2 + \frac{1}{2}(T+T^*)^2,$$

$$\begin{aligned} V_{\rm SUGRA} &= e^{K/M_P^2}\left[|D_S W|^2 + |D_T W|^2 - 3\frac{|W|^2}{M_P^2}\right] \\ &\approx \kappa^2 M^4\left(1+\frac{\phi^2}{M_P^2}\right)\left[\left|1-\sum_n \frac{m_n^2}{M^2}e^{-nT/f}\right|^2 + \frac{f^2|S|^2}{M_P^4}\left|\frac{\sqrt{2}\phi}{f} + \frac{M_P^2}{f^2}\sum_n n\frac{m_n^2}{M^2}e^{-nT/f}\right|^2\right] \\ &\approx \kappa^2 M^4\left[1-\left(\sum_n \frac{m_n^2}{M^2}e^{-nT/f} + \text{h.c.}\right) + \frac{\delta\phi^2}{M_P^2}\right], \end{aligned}$$

$\phi\ (\equiv \langle\phi\rangle + \delta\phi)$ obtains the Hubble induced mass: $\mu^4\delta\phi^2/M_P^2 = 3H^2\delta\phi^2$

"Natural-Hybrid Model"

During inflation, the waterfall fields are decoupled, and so the superpotential and Kahler potential are

$$W_{\rm inf} = \kappa S\left(M^2 - \sum_n m_n^2 e^{-nT/f}\right), \quad \text{and} \quad K_{\rm inf} = |S|^2 + \frac{1}{2}(T+T^*)^2,$$

$$\begin{aligned}
V_{\rm SUGRA} &= e^{K/M_P^2}\left[|D_S W|^2 + |D_T W|^2 - 3\frac{|W|^2}{M_P^2}\right] \\
&\approx \kappa^2 M^4\left(1+\frac{\phi^2}{M_P^2}\right)\left[\left|1-\sum_n \frac{m_n^2}{M^2}e^{-nT/f}\right|^2 + \frac{f^2|S|^2}{M_P^4}\left|\frac{\sqrt{2}\phi}{f} + \frac{M_P^2}{f^2}\sum_n n\frac{m_n^2}{M^2}e^{-nT/f}\right|^2\right] \\
&\approx \kappa^2 M^4\left[1-\left(\sum_n \frac{m_n^2}{M^2}e^{-nT/f} + \text{h.c.}\right)\right.
\end{aligned}$$

$$\langle\phi\rangle/f = \mathcal{O}(M_P^2 m_n^2/f^2 M^2) \ll 1$$

$\phi\ (\equiv \langle\phi\rangle + \delta\phi)$ obtains the Hubble induced mass: $\mu^4\delta\phi^2/M_P^2 = 3H^2\delta\phi^2$

"Natural-Hybrid Model"

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 {\rm sin}\frac{a}{f} - \lambda_2 {\rm sin}\frac{2a}{f} + \alpha {\rm log}\frac{\sigma}{\Lambda}\right),$$

where $\mu^4 \equiv \kappa^2 M^4$, $\lambda_{1,2} \equiv \pm 2im_{1,2}^2/M^2$ $(\ll 1)$, and $\alpha \sim \kappa^2/8\pi^2$ $(\ll 1)$

"Natural-Hybrid Model"

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda} \right),$$

where $\mu^4 \equiv \kappa^2 M^4$, $\lambda_{1,2} \equiv \pm 2 i m_{1,2}^2/M^2$ $(\ll 1)$, and $\alpha \sim \kappa^2/8\pi^2$ $(\ll 1)$

Only two terms among $e^{-nT/f}$
are enough to satisfy
All the Requirements.

We will show that all the requirements can be addressed only with the above terms.

"Natural-Hybrid Model"

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda} \right) ,$$

where $\mu^4 \equiv \kappa^2 M^4$, $\lambda_{1,2} \equiv \pm 2 i m_{1,2}^2/M^2$ $(\ll 1)$, and $\alpha \sim {}^2/8\pi^2$ $(\ll 1)$

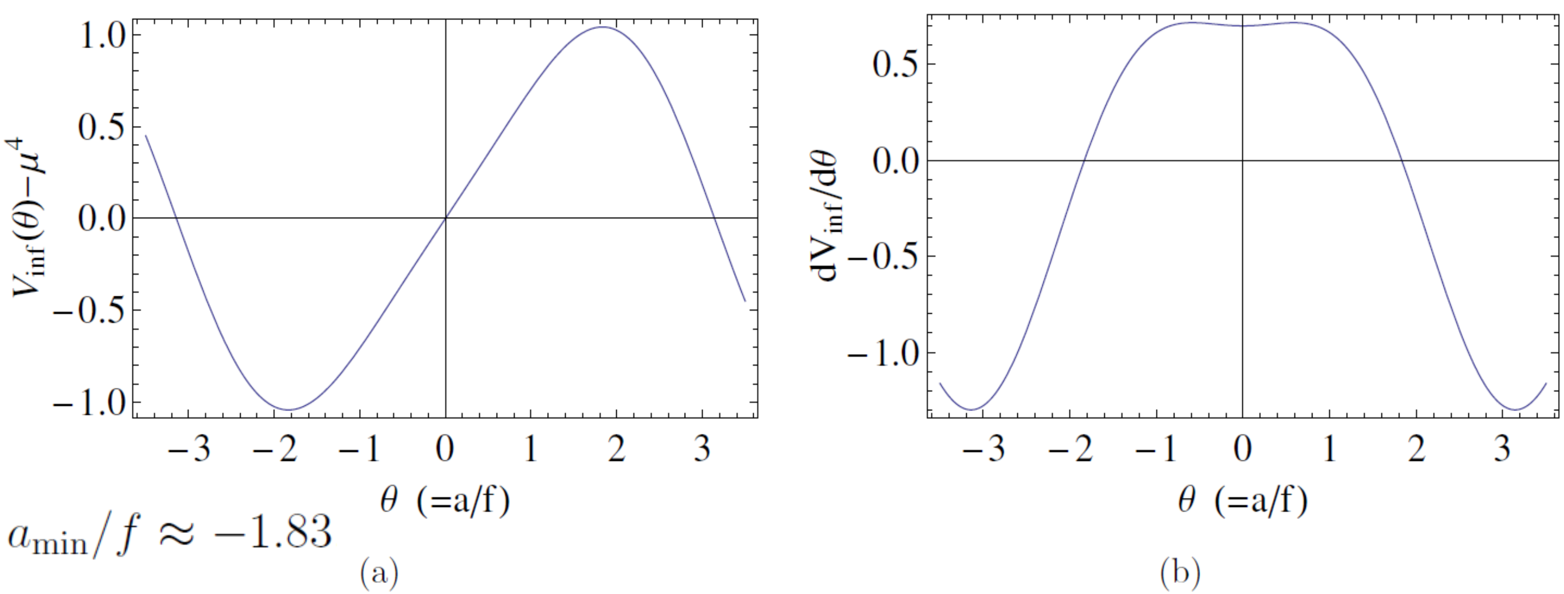
Quantum Correction generated when the heavy waterfall fields are integrated out

We will show that all the requirements can be addressed only with the above terms.

"Natural-Hybrid Model"

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda}\right),$$

$$\lambda_2/\lambda_1 = 0.15$$



$a_{\rm min}/f \approx -1.83$

"Natural-Hybrid Model"

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda}\right),$$

The slow-roll parameters are

$$\epsilon_a = \frac{M_P^2}{2}\frac{(\mu^8/f^2)(\lambda_1 \cos\frac{a}{f} - 2\lambda_2 \cos\frac{2a}{f})^2}{V_{\rm inf}^2} \approx \frac{\lambda_1 \xi^2}{2}\left(\cos\theta - 2\beta\ \cos 2\theta\right)^2,$$

$$\eta_a = \frac{M_P^2(\mu^4/f^2)(-\lambda_1 \sin\frac{a}{f} + 4\lambda_2 \sin\frac{2a}{f})}{V_{\rm inf}} \approx -\xi^2\left(\sin\theta - 4\beta\ \sin 2\theta\right),$$

$$\epsilon_\sigma = \frac{M_P^2}{2}\frac{\alpha^2\mu^8/\sigma^2}{V_{\rm inf}^2} \approx \frac{\alpha}{2\chi^2}, \qquad \eta_\sigma = -\frac{M_P^2\mu^4\alpha/\sigma^2}{V_{\rm inf}} \approx -\frac{1}{\chi^2},$$

$$\xi^2 \equiv \frac{M_P^2\lambda_1}{f^2}, \quad \beta \equiv \frac{\lambda_2}{\lambda_1}, \quad \text{and} \quad \theta \equiv \frac{a}{f}, \quad \chi \equiv \frac{\sigma}{\sqrt{\alpha}M_P}$$

“Natural-Hybrid Model”

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda}\right),$$

where $\mu^4 \equiv \kappa^2 M^4,\ \lambda_{1,2} \equiv \pm 2 i m_{1,2}^2/M^2\ (\ll 1)$, and $\alpha \sim \kappa^2/8\pi^2\ (\ll 1)$

In two field inflation,

$$\mathcal{P}_\zeta \approx \frac{\mu^4 \bar{u}^2}{24\pi^2 M_P^4 \epsilon_a^*}(1+\hat{r}),$$

$$n_\zeta - 1 \approx -2(\epsilon_a^* + \epsilon_\sigma^*) + 2\frac{-2\epsilon_a^* + \bar{u}^2(\eta_a^* + \eta_\sigma^* \hat{r})}{\bar{u}^2(1+\hat{r})},$$

$$r \approx \frac{16\epsilon_a^*}{\bar{u}^2(1+\hat{r})},$$

“ * ” denotes the values evaluated at a few Hubble times after horizon exit of inflation.

$\hat{r}$ is determined by the initial and end effects of inflation.

“Natural-Hybrid Model”

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda}\right),$$

where $\mu^4 \equiv \kappa^2 M^4$, $\lambda_{1,2} \equiv \pm 2 i m_{1,2}^2/M^2$ $(\ll 1)$, and $\alpha \sim \kappa^2/8\pi^2$ $(\ll 1)$

$$\hat{r} = \frac{\epsilon_a^*}{\epsilon_\sigma^*}\frac{\bar{v}^2}{\bar{u}^2}, \qquad \bar{u} \equiv \frac{R\epsilon_a^e}{\bar{\epsilon}^e} \quad \text{and} \quad \bar{v} \equiv \frac{\epsilon_\sigma^e}{\bar{\epsilon}^e}$$

$$\left(\bar{\epsilon}^e = R\epsilon_a^e + \epsilon_\sigma^e \right)$$

“ e ” denotes the values evaluated at the end of inflation.

$$R = \frac{\partial_\sigma V_e}{\partial_a V_e}\,\frac{\partial_a E}{\partial_\sigma E}.$$

parametrizes how much the hypersurface of the two inflatons are deviated from the hypersurface of the uniform energy density.

"Natural-Hybrid Model"

- One can show that $\hat{r} \ll 1$, only if $\epsilon_a^*/\epsilon_\sigma^* \lesssim 10^3$.
- Cosmological observables are dominated by dynamics of "a".
- e.g. with $\epsilon_a{}^* \approx 0.01$ ($\gg \epsilon_\sigma{}^* \approx \alpha/200$), $f \approx 0.8\, M_P$,
  $\sigma^*/M_P \approx 0.3$, $\kappa \approx 0.3$, $M \approx 2 \times M_G$,

  we can satisfy all the requirements.

wh

In tw

$$\mathcal{P}_\zeta \approx \frac{\mu^4 \bar{u}^2}{24\pi^2 M_P^4 \epsilon_a^*} (1+\hat{r}) \,,$$

$$n_\zeta - 1 \approx -2(\epsilon_a^* + \epsilon_\sigma^*) + 2\frac{-2\epsilon_a^* + \bar{u}^2(\eta_a^* + \eta_\sigma^* \hat{r})}{\bar{u}^2(1+\hat{r})},$$

$$r \approx \frac{16\epsilon_a^*}{\bar{u}^2(1+\hat{r})},$$

denotes the values evaluated at a few Hubble times after horizon exit of inflation.

$\hat{r}$ is determined by the initial and end effects of inflation.

“Natural-Hybrid Model”

$$V_{\rm inf} \approx \mu^4 \left(1 + \lambda_1 \sin\frac{a}{f} - \lambda_2 \sin\frac{2a}{f} + \alpha \log\frac{\sigma}{\Lambda} \right) ,$$

$$\Delta N_a \approx \frac{f^2}{M_P^2 \lambda_1} \int_{-1.0}^{0.4} \frac{d\theta}{\cos\theta - 2\beta\ \cos 2\theta} = \frac{1}{\xi^2} \int_{-1.0}^{0.4} \frac{d\theta}{\cos\theta - 2\beta\ \cos 2\theta} \approx 8.$$

$$N_\sigma = \frac{1}{M_P^2} \int_e^* \frac{V}{\partial V / \partial \sigma} d\sigma \approx \frac{1}{M_P^2} \int_e^* \frac{\sigma}{\alpha} d\sigma = \frac{1}{2} \left(\chi_*^2 - \chi_e^2 \right) = 50.$$

Conclusion

The large tensor spectrum of BICEP2 and sub-Planckian inflation require Two Field Inflation.

We have only a few ways for a naturally light inflaton.

Conclusion

The large tensor spectrum of BICEP2 and sub-Planckian inflation require Two Field Inflation.

1. SUSY hybrid + Natural = Natural hybrid Infl. and
2. SUSY hybrid + SUSY hybrid = Double hybrid Infl.

are successful.